Int. J. Heat Mass Transfer. Vol. 30, No. 6, pp. 1083–1093, 1987
Printed in Great Britain

0017-9310/87 $3.00+0.00
Pergamon Journals Ltd.

The details of the convective evaporation of dense and dilute clusters of drops

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(*Received* 19 *May* 1986 *and in final form* 23 *September* 1986)

Abstract—A global model describing the convective evaporation of dense and dilute clusters of drops has been formulated starting from first principles. The volume of the cluster and the number of drops in a given cluster are fixed and the drops do not move with respect to each other. The model has been tested for three different drag models and shows less than 10% sensitivity in the prediction of the droplet lifetime. A thorough parametric study has been performed and the results show that the control parameters are very different in order of importance for dense and dilute clusters. The initial relative velocity between drops and gases is a weak control parameter in the 40–1000 cm s^{-1} regime.

1. INTRODUCTION

EVAPORATION, ignition and combustion of fuel sprays in practical systems is influenced by many factors such as drop characteristics, surrounding gas temperature, composition and pressure, and the relative velocity between the spray and the gases. From the point of view of controlling evaporation, ignition and combustion, it is first very important to understand how the variation of each one of the above parameters modifies the phenomenon of interest, and second it is necessary to identify the regime(s) that is (are) most sensitive to the variation of a given parameter.

In some of our previous work [1] we have studied the influence of the above-mentioned parameters upon evaporation using a model that could equally describe quiescent evaporation of dense and dilute sprays. We have further developed a model of convective evaporation of non-dilute clusters of drops [2] that yielded results which, if confirmed experimentally, could be both used to simplify the models utilized to predict spray processes and also to understand the controlling parameters during convective spray evaporation. In particular, it was predicted that the initial relative velocity between a cluster of drops and the ambient flow is a weak control parameter because a large variation in its value results in relatively small changes in the evaporation time. The initial surrounding gas temperature was found to have a strong influence in the lower temperature regime, 750–1500 K, whereas in the higher temperature regime the influence was weak. Finally, it was also shown that the drop temperature was transient throughout the drop lifetime much as in the case of a single isolated drop [3], but unlike in the case of isolated drops [3], for dense clusters of drops, nonuniformities in the drop temperature persisted up to the first third of the total evaporation time at most. These results were obtained for a single-component fuel.

The present model is different from that of ref. [2] in that the new convective evaporation formulation pertains to any geometrical droplet-cluster shape and it is equally valid for dense and dilute clusters of drops. Thus, with this new model we have been able to identify important aspects of convective evaporation that pertain to the dense and dilute regimes.

Section 2 presents the new model for convective evaporation and the results obtained by solving the model equations are discussed in Section 3. Conclusions are given in Section 4.

2. PHYSICAL ANALYSIS AND MATHEMATICAL FORMULATION

The present analysis is for a monodisperse cluster of drops uniformly distributed within a gaseous volume, the ensemble of which is called V_c. The cluster volume is itself imbedded in a gaseous control volume, V, which is assumed self-similar to V_c. The gases outside the cluster are quiescent. The cluster of drops is injected into these quiescent gases at velocity u_d^0. As the drops move, the gases inside the cluster acquire a velocity u_g. For this reason the gases inside the cluster and outside the cluster are treated differently and have respective velocities of u_g and zero. The drops in the cluster do not move with respect to each other but instead they move through the gases as an entity at velocity u_d. Even though there is a net flow of gases and heat through the surface of the cluster, these effects are not modeled in detail and instead only their global influence is considered. As it will be seen later, this is a very good approximation for non-dilute clusters where penetration is confined to a very thin shell. On the other hand for dilute clusters where penetration will be shown to be substantial, the effect of this penetration will be appropriately taken into account by correlations relating the evaporation rate in convective flows to that in quiescent flow and to the Reynolds number. For intermediate regimes the model is still expected to be a good approximation of the global picture. No detailed fluid model is attempted here.

NOMENCLATURE

a	radius of the sphere of influence [cm]
A_c	transverse area of the cluster [cm^2]
A_d	cross-sectional area of a drop [cm^2]
C	non-dimensional evaporation rate, $-\dot{m}/(4\pi\rho_g D R^0)$
C_{Dc}	drag coefficient for the cluster
C_D	drag coefficient for one drop
C_p	specific heat at constant pressure [cal g^{-1} K^{-1}]
C_{pm}	arithmetic average of C_p evaluated at T_{gs} and T_{ga} [cal g^{-1} K^{-1}]
D	gas diffusivity [cm^2 s^{-1}]
F_{ev}	force of the evaporating gas [g cm s^{-2}]
F_{cev}	cluster-evaporation force [g cm s^{-2}]
F_T	surface force on V [g cm s^{-2}]
F_x	force which is due to small pressure gradients [g cm s^{-2}]
L	latent heat of evaporation [cal g^{-1}]
L_p	penetration distance [cm]
$\dot{m}$	evaporation rate [g s^{-1}]
m	mass [g]
N	total number of drops in V_c
n	drop number density in V_c [cm^{-3}]
r	radial coordinate [cm]
R	drop radius [cm]
R_1	non-dimensional drop radius, R/R^0
R_2	non-dimensional radius of the sphere of influence, a/R^0
Re	Reynolds number, $2Ru_r/\nu$
$\tilde{R}$	characteristic dimension of the cluster [cm]
T	temperature [K]
t	time [s]
u	velocity [cm s^{-1}]
V	control volume [cm^3]
V_c	volume of the cluster [cm^3]
x	coordinate along the cluster trajectory [cm]
Y	mass fraction
z	non-dimensional internal drop coordinate, r/R.

Greek symbols

θ	non-dimensional temperature, $C_{pg}T/L_{bn}$
μ	viscosity [g cm^{-1} s^{-1}]
ν	kinematic viscosity, μ/ρ [cm^2 s^{-1}]
ρ	density [g cm^{-3}]
$\phi(\phi_s)$	air/fuel mass ratio (stoichiometric)
$\phi'(\phi'_s)$	oxygen/fuel mass ratio (stoichiometric)
Λ	blowing number, $\rho_{gs}u_{evap}/(\rho_{ga}u_r)$.

Subscripts

a	at the edge of the sphere of influence
bn	at the normal boiling point
c	cluster
d	drop
evap	evaporation
F	fuel
g	gas
l	liquid
r	relative
s	drop surface
v	vapor.

Superscripts

0	initial condition.

The main assumptions regarding the liquid phase and the gas phase have been described in detail elsewhere [2] and thus will not be discussed here. What is of interest here, which will be discussed in detail, is the momentum transfer between drops and gases. The liquid-phase formulation and the evaporation law are treated in the same manner as in ref. [2] and the formulation of the quiescent-evaporation situation is that of ref. [1]. In that formulation [1], each drop of the cluster is surrounded by a sphere of influence whose radius is the half distance between the centers of two adjacent drops. The ensemble of these spheres of influence and the space between them constitutes V_c. Quiescent evaporation of each drop inside its own sphere of influence is described using quasi-steady diffusion differential equations for species and heat; these equations are coupled to the liquid phase equations through boundary conditions at the drop surface. The solutions of these differential equations are used in an integral set of equations for the entire cluster volume that describe heat, mass and species conservation. The global equation of state completes the formulation of the quiescent situation. To describe convective evaporation, the correlation of ref. [2] between the quiescent evaporation rate, the evaporation rate in convective flow and the Reynolds number will be used. The Reynolds number here is based upon the radius of each drop and the relative velocity $u_r = u_d - u_g$ ($u_d > u_g$ is assumed). The emphasis of the following model is upon the accurate calculation of u_r for both dense and dilute sprays.

We make the assumption that the momentum transfer between drops and gases can be treated on a local basis using, consistent with the global formulation, the assumption of small pressure gradients. Moreover, momentum exchange between the two phases is assumed to have three basic components: (i) transfer due to evaporation, (ii) transfer by fluid flow interaction in the form of a drag coefficient, and (iii) transfer due to small pressure gradients. This splitting allows the model to yield the correct limits in the cases of no evaporation ($\dot{m} \to 0$), no slip ($u_r \to 0$) and/or quiescent ambient gases ($u_g \to 0$).

Since volume forces are neglected in this model, the momentum equation for the control volume V is

$$\frac{\mathrm{d}}{\mathrm{d}t}[\rho_g u_g V + N m_d u_d] = F_T. \tag{1}$$

This yields

$$\rho_g V \frac{\mathrm{d}u_g}{\mathrm{d}t} + N\left[m_d \frac{\mathrm{d}u_d}{\mathrm{d}t} - \dot{m}(u_d - u_g)\right] = F_T \tag{2}$$

with

$$\dot{m} \equiv -\frac{\mathrm{d}m_d}{\mathrm{d}t} = \frac{1}{N}\frac{\mathrm{d}}{\mathrm{d}t}(\rho_g V). \tag{3}$$

The surface force F_T is due to any fluid dynamic deflection of the outer gas by the drop cluster acting as an entity. (Far from the cluster, the outer gas is quiescent.)

Now, on a local basis, inside V_c, the momentum equation for the gases is

$$\rho_g \frac{\mathrm{d}u_g}{\mathrm{d}t} = n\left[F_{ev} + \frac{1}{2}\rho_g A_d C_D (u_d - u_g)^2 + F_x\right] \tag{4}$$

where the right-hand side represents the interaction volume force of the drops acting on the gas. This force is split into three components according to assumptions (i)–(iii) above. Forces F_T, F_x and F_{ev} in equations (2) and (4) will be determined by requiring consistency with the three limits mentioned above.

In the limit $\dot{m} \to 0$, equation (4) is consistent if $F_{ev} \sim \dot{m}$ is assumed, according to the definition of F_{ev}.

The $u_g \to 0$ and no-slip limits can be studied from the equation for the relative velocity which is derived by subtracting from $\mathrm{d}u_d/\mathrm{d}t$, obtained from equation (2), $\mathrm{d}u_g/\mathrm{d}t$, obtained from equation (4)

$$\frac{\mathrm{d}u_r}{\mathrm{d}t} = \frac{1}{m_d}[F_T/N + \dot{m}u_r - (1 + nm_d/\rho_g)(F_{ev} + F_x)] - \frac{1}{2}n[1 + \rho_g/(nm_d)]A_d C_D u_r^2. \tag{5}$$

The $u_g = 0$ situation is taken as the initial condition. Physically, since only the drops are moving, $F_T = 0$ when $u_g = 0$, according to the definition of F_T. When $u_r \to 0$, $\mathrm{d}u_r/\mathrm{d}t \to 0$ and also $F_{ev} = 0$. This means that $F_{ev} \sim u_r$. Additionally, equation (5) yields in this limit

$$F_x = F_T/[N(1 + nm_d/\rho_g)]. \tag{6}$$

Equation (6) is interpreted as the influence of the surface force F_T upon the gas velocity through the pressure changes in V_c. Now, the no-slip limit represents the long time behavior for dense sprays where the entire cluster contained inside V_c acts as one big drop. Thus, in this limit it is expected that the momentum equation will have the following form

$$(\rho_g V_c + N m_d)\frac{\mathrm{d}u_c}{\mathrm{d}t} = F_{cev} - \frac{1}{2}\rho_g A_c C_{Dc} u_c^2 \tag{7}$$

where $u_c \equiv u_d = u_g$.

Assuming $F_{ev} = \lambda \dot{m} u_r$ and comparing term by term equation (7) with the drop momentum equation

$$m_d \frac{\mathrm{d}u_d}{\mathrm{d}t} = \dot{m}u_r(1 - \lambda) - \frac{1}{2}\rho_g A_d C_D u_r^2 + nm_d F_T/[N(\rho_g + nm_d)] \tag{8}$$

that has been obtained from a linear combination of equations (2) and (4), one concludes that

$$F_{cev} = \dot{m}_c u_d (1 - \lambda) \tag{9}$$

where $\dot{m}_c \equiv N\dot{m}$. But when $u_r \to 0$, from equations (7) and (8)

$$F_T = F_{cev} - \frac{1}{2}\rho_g A_c C_{Dc} u_c^2. \tag{10}$$

Since $F_T = 0$ for $u_g = 0$, this implies $\lambda = 1$. Then using equation (10), the simplest generalization for F_T is

$$F_T = -\frac{1}{2}\rho_g A_c C_{Dc} u_g u_d \tag{11}$$

because it satisfies both limits $u_g \to 0$ and $u_r \to 0$. This formula implies an effective cloud area of $A_c(u_g/u_d)$, the velocity ratio being equal to the non-slip displacement gas flux divided by the total gas flux. Also note that the choice $\lambda = 1$ is consistent with the literature [4, 5, 7, 9] which commonly represents the effect of evaporation (blowing) on drop drag as a reduction in the drag coefficient rather than an additional thrust term. Thus, the two momentum equations that will be solved here are

$$\frac{\mathrm{d}u_r}{\mathrm{d}t} = -n\left[\dot{m}u_r/\rho_g + \frac{1}{2}[1 + \rho_g/(nm_d)]A_d C_D u_r^2\right] \tag{12}$$

$$m_d \frac{\mathrm{d}u_d}{\mathrm{d}t} = -\frac{1}{2}\left[\rho_g A_d C_D u_r^2 + \frac{nm_d}{(\rho_g + nm_d)}\rho_g u_g u_d C_{Dc} A_c/N\right]. \tag{13}$$

Here C_{Dc} is based upon a Reynolds number using u_d and the length scale $[A_c(u_g/u_d)/\pi]^{0.5}$.

In general C_D depends upon the Reynolds number and the evaporation rate as well as R_2/R_1. However, here we neglect 'blockage' effects and thus C_D depends only upon Re and $\dot{m}$. In contrast, C_{Dc} depends only upon the Reynolds number. In order to assess the sensitivity of the model to the drag term, three different drag models will be compared. The first one is that of Eisenklam *et al.* [4] who correlated C_D to the drag coefficient in the absence of evaporation as follows:

$$C_D = C_{D_{\dot{m}=0}}/[1 + C_{pm}(T_{ga} - T_{gs})/L_{bn}]. \tag{14}$$

Here we take

$$C_{D_{\dot{m}=0}} = 24[1 + 0.0545Re + 0.1(1 - 0.03Re)\sqrt{Re}]/Re \tag{15}$$

which is valid for $1 \leqslant Re \leqslant 100$ [5]. A similar model has already been used in calculations of the ignition of dense sprays in convective flows [6].

The second drag coefficient model that will be used here is due to Yuen and Chen [7]. The experimental results of ref. [7] show, for a variety of liquids, that the classical drag expression is satisfied providing one uses the following Reynolds number

$$Re^* = \frac{\rho_{ga}(T_{ga})u_r 2R}{\mu_r(T_r)} \tag{16}$$

where the denominator is the gas viscosity evaluated at

$$T_r \equiv T_{gs} + (T_{ga} - T_{gs})/3 \tag{17}$$

using the formula of Wilke [8].

Finally, the recent results of the analysis of Cliffe and Lever [5] will be used to express C_D as a function of both the Reynolds number and the blowing number, Λ, which is defined in ref. [5] as the ratio of the radial gas velocity at the drop surface divided by u_r. Here we generalize this definition of Λ by also taking into account the ratio of the gas density at the surface to the external gas density. The drag coefficient is given in ref. [5] both in tabular and curve form for $1 \leqslant Re \leqslant 100$ and $0 \leqslant \Lambda \leqslant 3$. These results show significant departures both from the Stokes drag coefficient and the low Reynolds number solution of Dukowicz [9] as a function of Re and Λ. A curve fit of the tabular results of Cliffe and Lever [5] was used in the calculations to find the solutions for u_r and u_d from equations (12) and (13).

These equations, along with the liquid phase and the other gas phase equations, were solved using the Gear integrator. The algorithm for the solution has been described elsewhere [2] and thus will not be discussed here. We note that in contrast with the formulation of ref. [2] which was relevant to non-dilute sprays but was questionable for dilute sprays, the present model is inherently valid for both dense and dilute sprays.

An interesting feature that is an indication of how much the convective flow affects the evaporation of the drops is the extent of external flow penetration inside the cluster. This is described here by a penetration distance which is estimated in coordinates fixed with the cluster. In these new Lagrangian coordinates equation (12) becomes

$$u_r \frac{du_r}{dx} = -n\left[\dot{m}u_r/\rho_g + \frac{1}{2}[1 + \rho_g/(nm_d)]A_d C_D u_r^2\right]. \tag{18}$$

The relaxation distance L_p, which is here interpreted as a penetration distance, is calculated at each given time by identifying the location where $u_r = 0$. When time is fixed $\dot{m}$, R, ρ_g and Re are considered constant and thus

$$L_p = \frac{2\ln[1 + u_r^0\rho_g(\rho_g/m_d + n)A_d C_D/(2\dot{m}n)]}{(\rho_g/m_d + n)A_d C_D}. \tag{19}$$

The ratio of L_p divided by a characteristic cluster dimension is an indication of the important phenomenon controlling evaporation. If this ratio is larger than unity, evaporation is convection controlled, whereas if this ratio is very small compared to unity, evaporation is diffusion controlled. Between those values there exists an intermediary regime where both convection and diffusion are important.

Note that the concept of the penetration distance is used here only as a criterion to decide the regime in which the cluster evaporates and it is not part of the fluid dynamics model which is itself self-contained. That is to say that the same equations with the same boundary conditions are solved independent of the value of L_p.

3. DISCUSSION OF RESULTS

Interest is focused here on several aspects of drop-cluster convective evaporation. First, we want to study the sensitivity of the equations to the model chosen for C_D, as explained above. Second, we want to determine the influence of u_r^0 upon the evaporation of both dense and dilute sprays. Further, we will study the influence of T_{ga}^0, T_{gs}^0 and Y_{Fva}^0 upon convective evaporation. Finally, we would like to ascertain whether a transient, uniform internal-drop-temperature description is sufficient in the case of dense sprays, as different from dilute sprays. Initial evidence of this possibility has already been mentioned elsewhere [2], but the study was not conclusive because the model was questionable for dilute sprays and thus a reliable comparison could not be made.

Our calculations were performed for n-decane ($\phi_s = 15.7$) and the values used for the thermophysical properties are the same as those in Table 1 of ref. [2]. Also, throughout the calculations $R^0 = 2 \times 10^{-3}$ cm and the characteristic dimension of the cluster is taken to be $\tilde{R} = 10$ cm.

In all our calculations we have arbitrarily taken $u_r^0 = u_d^0$, although any other combination between u_r^0, u_d^0 and u_g^0 could be taken. The physical interpretation of the following results is thus related to the entrainment by and the relaxation of a cluster of evaporating drops introduced into quiescent gases.

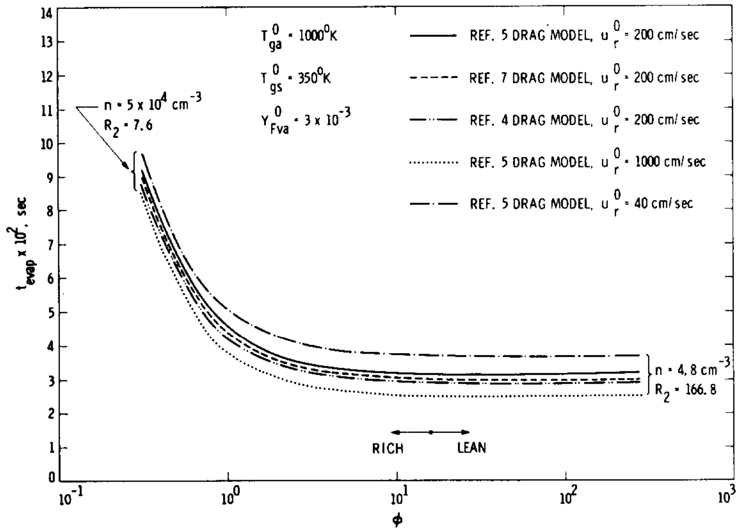


FIG. 1. Dependence of the evaporation time upon the air/fuel mass ratio for several drag models and several values of the initial relative velocity between the drops and the gases.

By taking $u^0_r < u^0_d$ one would study instead a situation where a cluster of drops is introduced into a convective flow. Obviously, many possibilities exist and we are here studying only one of them.

In Fig. 1 we display the evaporation time, defined as the time when $R_1 = 0.04$, vs the air/fuel mass ratio for the three drag models tested here. It is remarkable to notice how close the results are for all the drag models over the entire range of ϕ. In fact, if we had chosen a time scale such as to include the results from our richest case, $\phi = 0.157$, the three curves of Fig. 1 would have been indistinguishable. When the various evaporation times are compared for a given ϕ, the maximum uncertainty between the results is 10%. This is certainly as good or better than experimental uncertainties in the measurement of drag coefficients. Thus, we conclude that our model is insensitive to the drag model utilized for the calculations.

Further, in Fig. 1 we can see that similarly to quiescent evaporation [1], for convective evaporation, the evaporation time in the dense-cluster regime is larger than that in the dilute-cluster regime. As ϕ increases an asymptote is reached which corresponds to the situation of the isolated drop. Unlike the results previously presented that were the extension of a model applicable only to non-dilute sprays [2], we do not see here an increase in t_{evap} in the dilute cluster regime. In fact, when the plot of Fig. 2, which shows the penetration ratio vs R_1, is interpreted together with that of Fig. 1, one can see that as soon as a cluster of drops evaporates in the convective–diffusive regime or the convective regime, the evaporation time will be very close to that of an isolated drop exposed to the same initial conditions. This result is rather intuitive since, when they are present, convective effects always dominate diffusive effects during evaporation. For lower values of u^0_r diffusive-dominated evaporation becomes more important and thus the convective–diffusive and convective regimes are reached at larger values of ϕ, as shown in Fig. 3. The result is that the evaporation time reaches the isolated drop limit for larger values of ϕ, as shown in Fig. 1. We also notice, in agreement with the results of ref. [2], that u^0_r is a relatively weak parameter for the control of evaporation. However, whereas in ref. [2] this conclusion was obtained with a different model for non-dilute clusters of drops only, and inferred for

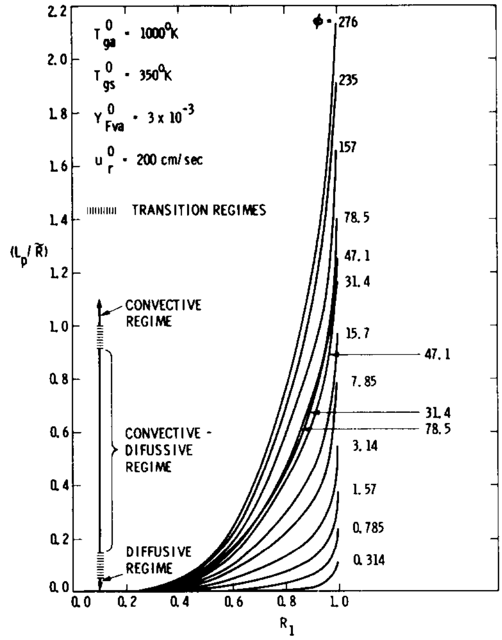


FIG. 2. Variation of the penetration ratio with the residual radius of the drops in a cluster, for clusters having various air/fuel mass ratios.

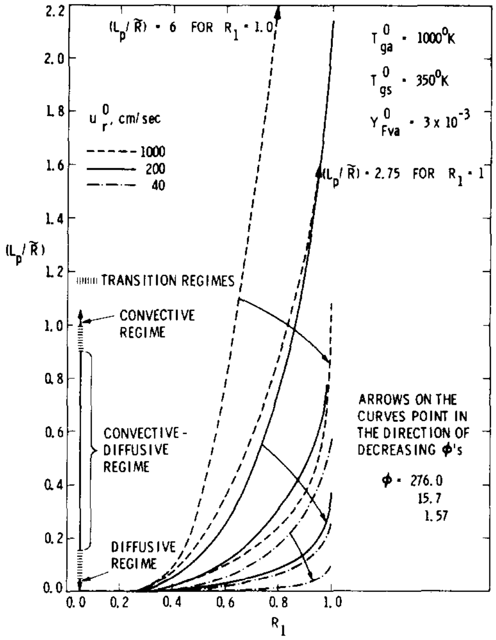


FIG. 3. Influence of the initial relative velocity upon the penetration ratio during evaporation of drops in clusters for three selected air/fuel mass ratios.

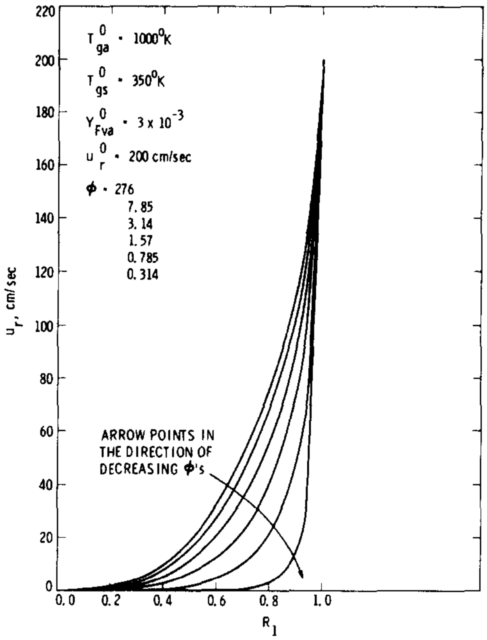


FIG. 4. Variation of the relative velocity between drops and gases with the residual drop radius for clusters of various air/fuel mass ratio.

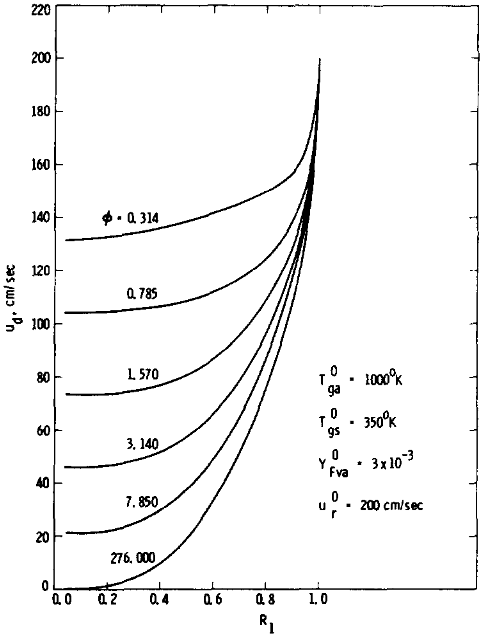


FIG. 5. Variation of the drop velocity with the residual drop radius for clusters of various air/fuel mass ratio.

the dilute clusters, here our conclusion holds over the entire range of ϕ. Figure 1 shows that by increasing u_r^0 by a factor of 25, t_{evap} can be decreased by at most 33%, this maximum being obtained for the most dilute cluster situation. In contrast, for the most dense cluster situation, t_{evap} decreases only by 15% when u_r^0 is increased by a factor of 25. The following results will show why the relative velocity is a weaker control parameter for dense clusters when compared to dilute clusters.

Since the time scale for evaporation varies with ϕ and since this is a two-phase problem where the interest is focused on the behavior of the evaporating drops, it is here relevant to scale the dependent variables with the residual radius of the drops rather than with time. In other words, starting with given initial conditions we look for trends that might be the same or different at fixed residual droplet mass. Plots of the same quantity vs time, while being interesting, would not provide the same insight into the physics of the situation.

The plots of the dependent variables shown in Figs. 4–7 are useful in building a better understanding of the differences between the evaporation of a dense cluster and that of a dilute cluster of drops. Figure 4 shows that the relative velocity of a dense cluster of drops decreases faster as a function of R_1 than that of a dilute cluster of drops and Fig. 5 shows that the opposite is true for the drop velocity. The reason for this is that when a denser cluster of drops moves through gases it exposes a greater surface area to the flow because the number of drops is larger and thus there is a stronger interaction between the gases and the drops. The result is a faster relaxation as shown in Fig. 4. In contrast, the drop velocity depends on the inertial effect of the cloud. Because a denser cloud has a higher mass it slows down less than a more dilute cloud.

As expected, the Reynolds number variation shown in Fig. 6 is similar to the variation of u_r. In agreement with the results of Dwyer and Sanders [10] for

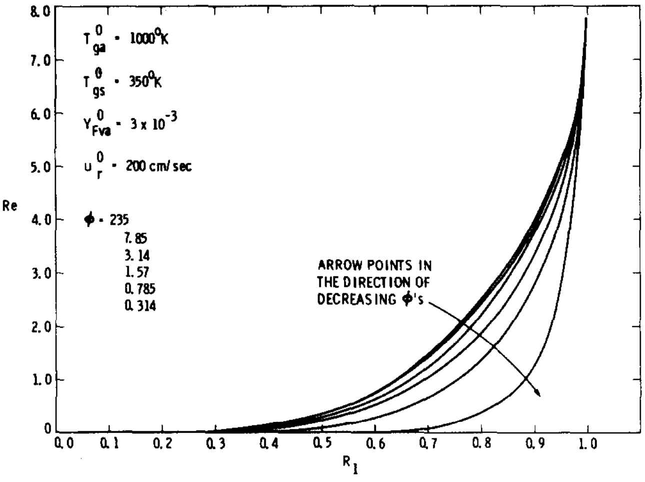


FIG. 6. Variation of the Reynolds number with the residual drop radius for clusters of various air/fuel mass ratio.

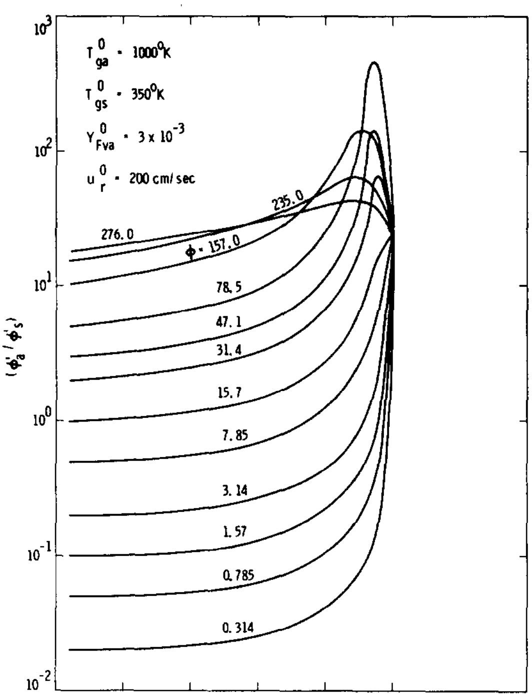


FIG. 7. Variation of the oxygen/fuel equivalence ratio with the residual drop radius for clusters of various air/fuel mass ratio.

isolated drops, we find here that the Reynolds number decreases very fast. Table 1 shows, for a selected range of cluster drop densities, the residual mass and the fraction of the total evaporation time when $Re = 1$ is reached during evaporation.

Consistent with the results discussed above, the oxygen/fuel mass equivalence ratio, displayed in Fig. 7 vs R_1, is larger at the beginning of evaporation and decreases as more fuel vapor is added to the gases. For the fuel-rich ($\phi < 15.7$) and the dense cluster cases, both the above effect and the fact that the relative velocity decreases fast result in a situation where the drop evaporates most of its lifetime in a fuel-rich environment. At stoichiometric ($\phi = 15.7$) and lean ($\phi > 15.7$) conditions the drops evaporate during their entire lifetime in an oxygen-rich environment. These observations are very important in terms of the determination of the ignition time for clusters of drops [6].

Table 1. Conditions at $Re = 1$ for various air/fuel mass ratios. Initial conditions: $T^0_{ga} = 1000$ K; $T^0_{gs} = 350$ K; $Y^0_{Fva} = 3 \times 10^{-3}$; $u^0_r = 200$ cm s^{-1}

ϕ	n (cm^{-3})	$t(Re = 1)/t_{evap}$	$m_d(Re = 1)/m^0_d$
276.0	4.8	0.50	0.27
1.57	9.9×10^3	0.47	0.33
0.314	5.0×10^4	0.11	0.68

Figure 8 shows the variation of the evaporation time with T^0_{ga}. From the plots displayed on the figure it is obvious that both curves present the same trends (i.e. t_{evap} decreases with increasing T^0_{ga} in the lower T^0_{ga} regime followed by an asymptotic behavior as T^0_{ga} increases further); however, for dense clusters the curve is shifted towards larger times and a higher T^0_{ga} regime. The shift in the position of the curves as one moves to denser clusters is due to the higher thermal inertia of the cluster. In the lower T^0_{ga} regime the decrease of t_{evap} with increase in T^0_{ga} is intuitively well understood because a larger T^0_{ga} implies a higher heat transfer to the drops which enhances evaporation, thereby reducing the lifetime of the drop. In the higher T^0_{ga} regime, because of the larger ambient temperature the gas density is lower, and since the pressure is the same in all cases, this means that heat transfer to the drops will also be the same. This explains the asymptotic behavior in the high T^0_{ga} range.

The results obtained for $T^0_{ga} = 2000$ K were also

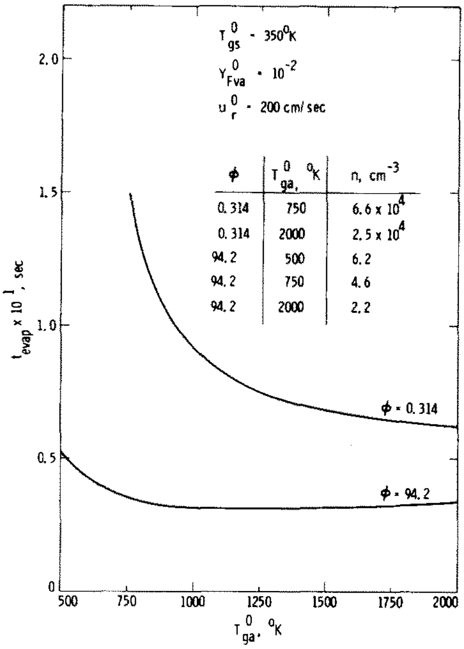


FIG. 8. Dependence of the evaporation time upon the initial surrounding gas temperature for a dense and a dilute cluster of drops.

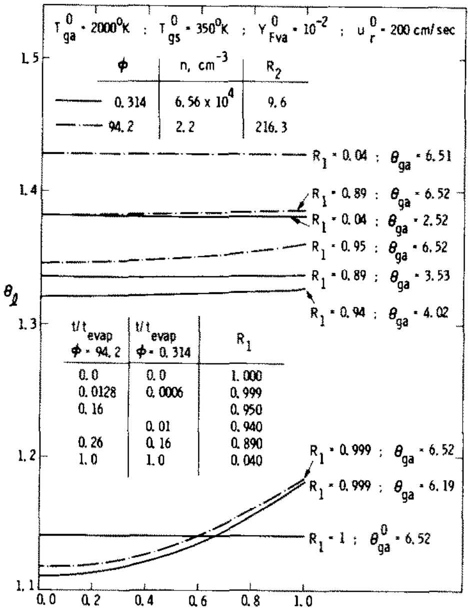
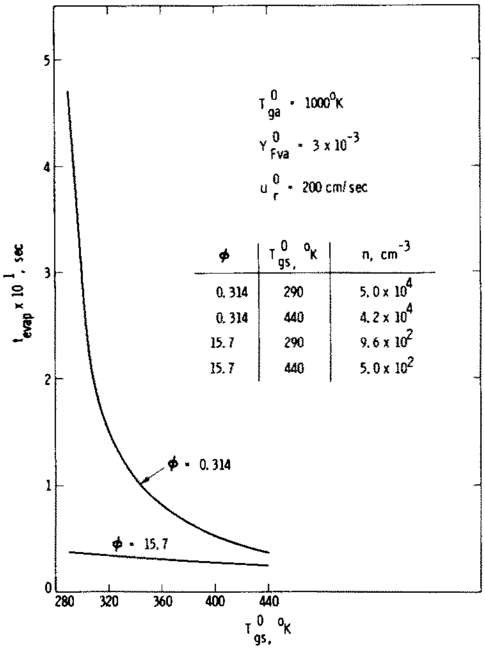


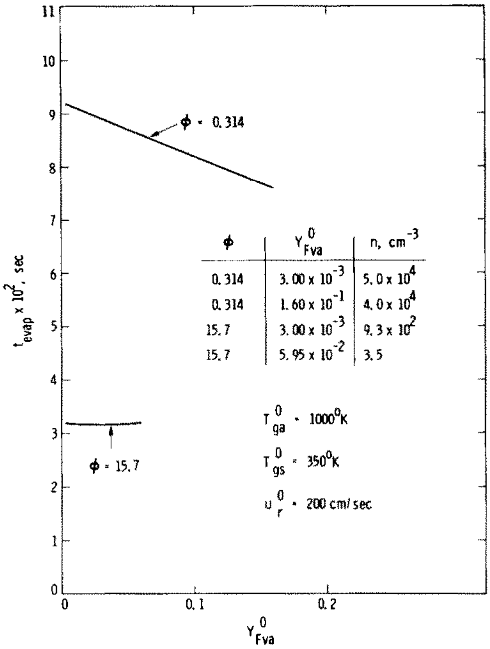
FIG. 9. Variation of the non-dimensional internal drop temperature with a non-dimensional radial coordinate at various residual drop sizes for a dense and a dilute cluster of drops.

used to investigate the evolution of the internal temperature profile of the drop for both dense and dilute clusters of drops. In this case $T^0_{ga} - T^0_{gs}$ is the largest, thereby inducing the largest initial temperature gradients inside the drop. The initial drop temperature is arbitrarily assumed uniform. Since this is not necessarily a solution of the heat conduction equation with the associated boundary conditions [2], a non-uniform profile at $R_1 = 0.999$ that now satisfies both the conduction equation and the boundary conditions can in fact be considered as the true initial conditions. We notice that the viscosity of the fuel is here large enough to make the zero-Hill-vortex-strength limit acceptable [3] because the ratio of the characteristic time for circulation to the characteristic time for heat-up, $[(\lambda_l/(\rho_l/C_{pl})]/(\mu_l/\rho_l)]$, is 3.53×10^{-2} (values of the properties are given in Table 1 of ref. [2]). Thus the heating time is independent of the circulation time and the use of the spherically symmetric heat conduction equation is justified. What is of interest here is to compare our results with those of ref. [3] in which it was found using a finite-Hill-vortex-strength model that for an isolated single-component fuel droplet the internal drop temperature was transient and nonuniform throughout the droplet lifetime. Thus we will be looking here at the relaxation time for θ_l in the case of dense and dilute sprays and comparing it with the results of ref. [3]. Because of our somewhat artificial initial condition we focus here only on relative and qualitative behavior. The non-dimensional drop temperature is plotted in Fig. 9 vs an internal, non-dimensional, drop coordinate at various residual drop sizes for both a dense and a dilute cluster. These plots show that the internal temperature is transient throughout the drop lifetime for both dense and dilute clusters. However, in the case of the dense cluster the temperature becomes uniform extremely fast whereas in a dilute cluster the drops experience a non-uniform temperature relatively for a longer time. For the situations studied here the internal drop temperature becomes uniform in the case of the dense cluster when $t/t_{evap} = 1.0 \times 10^{-2}$ and the residual mass is 83% whereas for a dilute cluster the corresponding numbers are 2.6×10^{-1} and 70%. We conjecture that the reason for this is that the evaporation rate of a dense cluster is slower [e.g. for $R_1 = 0.54$ the ratio $C(\phi = 0.314)/C(\phi = 94.2)$ is 0.56] and since non-uniformities in θ_l are the result of a competition between rate of heat conduction and rate of evaporation, the former now becomes faster than the latter, yielding a uniform θ_l. Thus, according to these results, for the purpose of studying dense clusters of single-component drops one could utilize a uniform internal drop-temperature model without any penalty in accuracy. Further studies must address the question of the penalty, if any, for using a uniform internal drop temperature in the case of dilute clusters. Although our results for dilute clusters present trends different from those of ref. [3] regarding the relaxation time for θ_l to a uniform condition, in agreement with ref. [3] we found that θ_l is transient up to the end of the droplet lifetime. However, the rate of increase in θ_l during the latter part of the drop's lifetime is rather small and sometimes negligible compared to its value during the initial portion of droplet evaporation.



ϕ	T^0_{gs}, °K	n, cm^{-3}
0.314	290	5.0×10^4
0.314	440	4.2×10^4
15.7	290	9.6×10^2
15.7	440	5.0×10^2

FIG. 10. Dependence of the evaporation time upon the initial drop temperature for a dense and a dilute cluster of drops.



ϕ	Y^0_{Fva}	n, cm^{-3}
0.314	3.00×10^{-3}	5.0×10^4
0.314	1.60×10^{-1}	4.0×10^4
15.7	3.00×10^{-3}	9.3×10^2
15.7	5.95×10^{-2}	3.5

FIG. 11. Dependence of the evaporation time upon the initial mass fraction of fuel vapor in the gas phase for a dense and a dilute cluster of drops.

Results showing the variation of t_{evap} with T^0_{gs} are presented in Fig. 10 for two air/fuel mass ratios. For dense clusters the thermal inertia of the liquid phase is larger than in the case of dilute clusters and thus it takes longer to heat up and evaporate drops starting at the same initial drop temperature. As the normal boiling point is approached the two curves converge, as expected, because then the evaporation time depends basically upon the evaporation rate rather than the heating rate. It is worthwhile noting that we find here a strong dependence of t_{evap} upon T^0_{gs} for dense clusters in agreement with the results of ref. [2] which were obtained with a different convective evaporation model, and a weak, linear, dependence for dilute clusters. The reason for this is that, in contrast to the dense cluster situation, in the case of the dilute cluster evaporation is mostly convection controlled or convection–diffusion controlled and thus t_{evap} is close to the isolated drop value.

In Fig. 11 we display results showing the variation of the evaporation time with Y^0_{Fva} for dense and dilute clusters of drops. For the dense cluster case ($\phi = 0.314$), Y^0_{Fva} is increased up to the value where saturation is encountered at the initial condition whereas for the dilute cluster case Y^0_{Fva} is increased until n becomes of the order of $1\ cm^{-3}$. Again, in agreement with our previous results [2] obtained for non-dilute sprays with a different convective evaporation model, we find here a linear decrease in t_{evap} with increasing Y^0_{Fva}. This is easily understood since for fixed ϕ, as Y^0_{Fva} increases, n decreases, so that for the same initial temperature and velocity conditions the thermal mass of the cluster is lower. Since evaporation occurs mostly in the diffusion-controlled regime according to the results of Figs. 1 and 2, t_{evap} is shorter. The situation is different for dilute sprays where evaporation occurs mostly in the convection-controlled regime and thus the evaporation time is close to the individual drop value. For this reason, t_{evap} is rather insensitive to Y^0_{Fva} for dilute clusters.

4. SUMMARY AND CONCLUSIONS

The model of convective drop-cluster evaporation developed here is inherently valid for both dense and dilute clusters and thus it is well suited for the comparative study of droplet evaporation in these two different configurations. As the air/fuel mass ratio increases from values for very dense to those for very dilute clusters, several distinctive evaporation regimes are identified. Very dense clusters were found to evaporate in a diffusion-controlled regime whereas very dilute clusters were found to evaporate in a convection-controlled regime. Between these two regimes there is a convection–diffusion regime where both phenomena are important in determining the lifetime of the cluster. These results were found to be insensitive to the drag model used. In fact, results obtained with three different drag models showed less than 10% variation in the evaporation time.

A parametric study has been performed in order to identify important parameters controlling drop-cluster evaporation in different regimes. Thus, it was found that the initial relative velocity between the gases and drops is a weak parameter both in the dilute and dense regimes. Furthermore, it has been found that the initial ambient surrounding temperature is a strong control parameter in the lower values of the T_{ga}^0 regime and a weak control parameter in the higher values of the T_{ga}^0 regime. Although the variation of the lifetime of the cluster with changes in initial ambient temperature is the same for dense and dilute clusters, because of the larger thermal inertia of the dense cluster, the corresponding curve is substantially shifted towards larger times and higher temperatures. Moreover, the study of the sensitivity to variations in the initial ambient temperature was also used to assess the duration of the relaxation time for the internal droplet temperature to a uniform condition. Given uncertainties due to the fact that we started with an arbitrary, uniform internal drop temperature that was not a solution of our equations, and that very shortly became nonuniform as a result of now satisfying our equations and boundary conditions, and that what we followed was the relaxation of this temperature profile, our conclusions are that the relaxation time is short for dilute clusters and very short for dense clusters. However, transients persist during most of the drop lifetime.

The initial drop temperature has been found to be, for dense clusters, a strong control parameter in the lower T_{ga}^0 regime and a somewhat weaker parameter in the higher T_{ga}^0 regime. In contrast to dense clusters, for dilute clusters the initial drop temperature is a very weak parameter in the entire regime of interest. As the initial drop temperature approaches the normal boiling point both curves for dense and dilute curves converge. These effects are strongly related to the thermal inertia of the spray.

Similarly, it has been found that for a fixed air/fuel mass ratio, the evaporation time of a cluster of drops is very insensitive to the initial mass fraction of fuel vapor in the ambient gases if the cluster is dilute and that the evaporation time decreases linearly with increasing Y_{Fva}^0 for dense clusters. The latter effect is due to the fact that as ϕ is fixed and Y_{Fva}^0 increases, less liquid fuel is present initially in the cluster.

To summarize, for dense sprays the control parameters are in order of importance: the initial drop temperature, mostly in the low regime; the initial surrounding gas temperature in the low regime; the initial mass fraction of fuel vapor in the gas phase; and the initial relative velocity between drops and gases. In contrast, for dilute sprays the control parameters are in order of importance: the initial temperature of the surrounding gas in the very low regime; the initial relative velocity between drops and gases; the initial drop temperature; and the initial mass fraction of fuel in the gas phase.

Acknowledgements—The research described in this paper was performed by the Jet Propulsion Laboratory, California Institute of Technology, and was supported by the Energy Conversion and Utilization Technologies Division, U.S. Department of Energy, and the U.S. Army Research Office through interagency agreements with the National Aeronautics and Space Administration.

REFERENCES

1. J. Bellan and R. Cuffel, A theory of non dilute spray evaporation based upon multiple drop interactions, *Combust. Flame* **51**(1), 55–67 (1983).
2. J. Bellan and K. Harstad, Analysis of the convective evaporation of non-dilute clusters of drops, *Int. J. Heat Mass Transfer* **30**, 125–136 (1987).
3. S. Prakash and W. A. Sirignano, Theory of convective droplet vaporization with unsteady heat transfer in the circulating liquid phase, *Int. J. Heat Mass Transfer* **23**, 253–268 (1980).
4. P. Eisenklam, S. A. Arunachalam and J. A. Weston, Evaporation rates and resistance of burning drops, 11th Symposium (International) on Combustion, The Combustion Institute, pp. 715–728 (1967).
5. K. A. Cliffe and D. A. Lever, Isothermal flow past a blowing sphere, *Int. J. Numer. Meth. Fluids* **5**, 709–725 (1985).
6. J. Bellan and K. Harstad, Ignition of non dilute clusters of drops in convective flows, submitted for publication.
7. M. C. Yuen and L. W. Chen, On drag of evaporating liquid droplets, *Combust. Sci. Technol.* **14**, 147–154 (1976).
8. C. R. Wilke, A viscosity equation for gas mixtures, *J. chem. Phys.* **18**, 517 (1950).
9. J. K. Dukowicz, An exact solution for the drag of a sphere in low Reynolds number flow with strong uniform blowing or suction, *Physics Fluids* **25**, 1117–1118 (1982).
10. H. A. Dwyer and B. R. Sanders, Detailed computation of unsteady droplet dynamics, presented at the 20th Symp. Int. on Combustion, 12–17 August (1984).

DETAILS DE L'EVAPORATION CONVECTIVE DE GRAPPES DENSES OU DILUEES DE GOUTTES

Résumé—Un modèle global décrivant l'évaporation convective de grappes denses ou diluées de gouttes a été formulé à partir des premiers principes. Le volume de la grappe et le nombre de gouttes dans une grappe sont fixés et les gouttes ne se déplacent pas les unes par rapport aux autres. Le modèle a été essayé pour trois modèles différents de trainée et il montre une sensibilité inférieure à 10% dans le calcul de la durée de vie de la gouttelette. Une étude paramétrique est conduite et les résultats montrent que le paramètres de contrôle sont très différents en ordre d'importance pour les grappes denses et diluées. La vitesse initiale relative entre gouttes et gaz est un paramètre peu important dans le domaine 40–1000 cm s^{-1}.

DIE EINZELHEITEN DER OBERFLÄCHENVERDAMPFUNG VON DICHTEN UND GELÖSTEN TROPFENVERBÄNDEN

Zusammenfassung—Ausgehend von den Grundgleichungen wurde ein umfassendes Modell zur Beschreibung der Oberflächenverdampfung von dichten und gelösten Tropfenverbänden formuliert. Das Verbundvolumen und die zahl der tropfen in einem gegebenen verbund sind fest, die tropfen bewegen sich relativ zueinander nicht. Das Modell wurde mit drei verschiedenen Reibungsmodellen getestet, wobei sich ein Einfluß von weniger als 10% auf die vorausberechnete Lebensdauer der Tropfen ergab. Eine sorgfältige Parameterstudie wurde durchgeführt. Die Ergebnisse zeigten, daß die den Vorgang kontrollierenden Parameter für dichte und gelöste Verbände sehr unterschiedlich hinsichtlich ihrer Wichtigkeit sind. Die Anfangs-Relativgeschwindigkeit zwischen den Tropfen und dem Gas ist im Bereich 40–1000 cm s^{-1} von geringem Einfluß.

ОСОБЕННОСТИ КОНВЕКТИВНОГО ИСПАРЕНИЯ ПЛОТНЫХ И РАЗРЕЖЕННЫХ КЛАСТЕРОВ КАПЕЛЬ

Аннотация—Сформулирована обобщенная схема конвективного испарения плотных и разреженных кластеров капель. Объем кластера и количество капель в нем заданы, капли неподвижны относительно друг друга. Такая схема проверена для трех различных моделей сопротивления, показано, что ошибка при оценке времени жизни капли составляет менее 10%. Результаты показывают, что определяющие параметры, которые существенны для данного процесса, различны по порядку величины для плотных и разреженных кластеров. Начальная относительная скорость капель и газов слабо влияет на результаты в диапазоне 40–1000 см. с$^{-1}$.